

Evolution of structure functions in a chiral potential model

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Abstract

Evolution of structure functions is studied in a chiral potential model which incorporates the pion as the chiral symmetry restoring field. Evolution equations for the quark and pion densities are derived in the lowest order at large Q^2 . The splitting function for quark emission from point-like pion is flavor-dependent. This is shown to lead to nontrivial evolution of the nonsinglet moments in the next order which is consistent with the observed departure from the Gottfried sum rule.

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1 Introduction.

The observed departure from the Gottfried sum rule is difficult to explain within the framework of perturbative QCD [1]. The effect indicates a flavor-asymmetry in the sea quark distribution inside a nucleon which cannot be generated by the Altarelli-Parisi evolution of the structure functions. The simplest alternative is to bring in pions into the picture [2,3,4] and replace gluon emission from a quark by pion emission [5]. Just having pions inside the nucleons cannot, of course, explain the observed effect because the direct contribution of the pion density drops out from the equation of evolution of the nonsinglet moment which is the object of interest in the context of the Gottfried sum. A rather complete treatment of the problem has been given by Ball and Forte [6] who have computed the non-perturbative cross sections for the emission of pseudoscalar bound states from quarks and obtained a satisfactory explanation of the observed phenomenon. Their work shows that the effect is essentially nonperturbative. Also, there is good theoretical ground to expect such evolution, even if the analysis of the experimental data is clouded with uncertainties.

In this paper we are interested in seeing whether such an evolution can result in the phenomenological models of the nucleons which have been devised to study the static properties of the nucleon bound state. The question is nontrivial since the connection between these models and the QCD-evolved structure functions is not at all obvious. On the other hand, the models of the nucleon bound state are supposed to be phenomenological manifestations of the nonperturbative QCD at the hadronic scale [16]. It is therefore natural to try to see to what extent the models serve as effective theories and whether they can be put to any use other than calculation of static properties of the nucleon. As an example, we have chosen in this paper a chiral potential model incorporating point-like pions and have calculated the relevant cross sections and the splitting functions using the quark-pion coupling of that model. The choice of this particular model is not due to its having any greater intrinsic merit than other existing models, but due to the simplicity of its quark-pion hamiltonian.

We report here only the low order perturbation results at large Q^2 and show that a nontrivial evolution of the nonsinglet distribution can indeed result from this model.

2 The model.

We consider, with some modification, a phenomenological independent-quark model of baryons studied by Jena and Panda [6,7] (see also [8-15]) who could obtain the static electromagnetic properties of the nucleon, the axial vector coupling constant in beta decay and the pion-nucleon coupling constant in reasonable agreement with the experiments. The quarks in this model move independently in a confining potential $V_q(\vec{r})$ which is an equal mixture of a vector and a scalar part:

$$V_q(\vec{r}) = \frac{1}{2}(1 + \gamma^0)V(\vec{r}) \quad (2.1)$$

with

$$V(\vec{r}) = a^2 r + V_o, \quad a > 0$$

The non-invariance of the quark mass term and the scalar part of $V_q(\vec{r})$ under SU(2) chiral transformation is sought to be adjusted by introducing a pion field interacting with the quarks through a linearised term in the lagrangian density

$$\mathcal{L}_I^\pi(x) = \frac{i}{f_\pi} G(\vec{r}) \bar{\psi}_q(x) \gamma^5 \tau \cdot \phi(\mathbf{x}) \psi_q(x) \quad (2.2)$$

$$G(\vec{r}) = m_q + \frac{V(\vec{r})}{2} \quad (2.3)$$

m_q is the quark mass and f_π is the pion decay constant. The quark wavefunction of the baryonic core is determined from $V_q(\vec{r})$; in the ground state configuration

$$\psi_q(\vec{r}) = N_q \begin{pmatrix} \phi_q(r) \\ \frac{-i\sigma \cdot \hat{r}}{\lambda_q} \frac{d\phi_q(r)}{dr} \end{pmatrix} \chi^\uparrow \quad (2.4)$$

$\phi_q(r)$ turns out to be expressible in terms of the Airy function.

The static properties of the baryons calculated from $V_q(\vec{r})$ alone (with $a \simeq 0.343$ GeV, $V_o \simeq -0.506$ GeV) are already close to the observed values so that, on a phenomenological level, the quark-pion coupling is expected to be small making perturbative calculations meaningful. On the other hand, this is an example of a potential model which allows one to calculate pion emission and absorption by quarks and their effect on the structure functions. We show this by considering, as the custom is, a quark beam which is being probed by a virtual photon, obtaining the splitting functions from the lowest-order cross sections.

The relevant cross sections are (a) $\sigma(\gamma^* q \rightarrow q' \pi)$, (b) $\sigma(\gamma^* q \rightarrow \pi q')$ and (c) $\sigma(\gamma^* \pi \rightarrow q \bar{q})$.

The processes (a) and (b) contain emission of quarks from quarks and pions, and lead to the splitting functions P_{qq} and $P_{q\pi}$. The process (c) gives the splitting function $P_{\pi q}$.

The external potential G having a long range part in the original model of Panda and Jena necessitates the introduction of an infrared cutoff parameter λ in the three momentum of the static gluon. Physically, this is fixed by the size of the baryons. With this modification, the effective $qq\pi$ vertex factor for perturbation calculations in this model becomes

$$\Gamma_{qq\pi}(p, p') = \frac{1}{2f_\pi} \left[V_o - \frac{4a^2(|\vec{p} - \vec{p}'| - \lambda)}{\pi\lambda|\vec{p} - \vec{p}'|} \right] \gamma^5 \quad (2.5)$$

where p and p' are the quark four-momenta going into and out of the vertex.

3 Virtual photon cross sections in the large Q^2 limit.

The evolution of quark and pion densities in this model can be obtained by first calculating the virtual photon cross sections listed in the previous section. The lowest order perturbation calculation using the vertex factor(2.5) is straightforward in the small transverse momentum and large Q^2 limit neglecting the quark and pion masses.

In the following p, p', k, q denote the (center-of-mass) four-momenta of the quarks q, q' , the pion and the photon. For the process(a), we define the invariant variables $s_a = (q + p)^2$, $t_a = (p' - q)^2$, $u_a = (p - p')^2$. Then in the leading order, the cross section $\sigma(\gamma^* q \rightarrow q' \pi)$ is

$$\frac{d\sigma}{d\Omega'} = \frac{2\pi\alpha}{s_a^2} e_{q'}^2 \left(\frac{-s_a}{t_a} - \frac{2(s_a + Q^2)Q^2}{s_a t_a} \right) \left[V_\circ - \frac{4a^2}{\pi\lambda} \right]^2 \quad (3.1)$$

Defining $s_b = (q + p)^2$, $t_b = (q - k)^2$ and $u_b = (p' - q)^2$, the cross section for the process $\gamma^* q \rightarrow \pi q'$ is

$$\frac{d\sigma}{d\Omega_\pi} = \frac{2\pi\alpha}{s_b^2} \frac{e_{q'}^2}{4f_\pi^2} \left[V_\circ - \frac{4a^2}{\pi\lambda} \right]^2 \frac{2Q^2}{-t_b} \quad (3.2)$$

Defining $s_c = (q + k)^2$, $t_c = (q - p')^2$ and $u_c = (p' - k)^2$, we find, to leading order in Q^2 , the cross section for the process $\gamma^* \pi \rightarrow q' \bar{q}$

$$\frac{d\sigma}{d\Omega'} = \frac{2\pi\alpha}{s_c^2} \frac{e_{q'}^2}{f_\pi^2} \left[V_\circ - \frac{4a^2}{\pi\lambda} \right]^2 \frac{1}{2} \left[\left(\frac{u_c}{t_c} \right) + \frac{2s_c Q^2}{t_c u_c} \left(\frac{e_\pi}{e_{q'}} \right)^2 - 1 \right] \quad (3.3)$$

To facilitate the extraction of the splitting functions, we express these cross sections in terms of the kinematic variables z and p_T for each process which are related to the momentum variables by

$$z \simeq \frac{Q^2}{s + Q^2} \quad (3.4)$$

$$p_T^2 \simeq \frac{-st}{s + Q^2} \quad (3.5)$$

the relations being true for small t and large Q^2 . Under these approximations,

$$\frac{d\sigma(\gamma^* q \rightarrow q' \pi)}{dp_T^2} = e_{q'}^2 \sigma_\circ \frac{\alpha_\pi}{p_T^2} \left[\frac{1 + z^2}{1 - z} \right] \quad (3.6)$$

$$\frac{d\sigma(\gamma^* q \rightarrow \pi q')}{dp_T^2} = e_\pi^2 \sigma_\circ \frac{\alpha_\pi}{p_T^2} 2z \quad (3.7)$$

$$\frac{d\sigma(\gamma^* \pi \rightarrow q' \bar{q})}{dp_T^2} = e_{q'}^2 \sigma_\circ \frac{\alpha_\pi}{p_T^2} \frac{1}{2} \left[(z^2 + (1 - z)^2) + \left(\frac{e_\pi^2}{e_{q'}^2} \right) z(1 - z) \right] \quad (3.8)$$

where $\sigma_\circ = 4\pi^2\alpha/s$ and

$$\alpha_\pi = \frac{1}{2\pi f_\pi^2} \left[V_\circ - \frac{4a^2}{\pi\lambda} \right]^2 \quad (3.9)$$

which is an effective coupling constant of the model in the limit of large Q^2 .

4. The evolution equations and splitting functions.

Integrating (3.6)-(3.8) with respect to p_T^2 (with an infra-red cutoff), we can now easily get the evolution of the structure function $F_2(x, Q^2)$. The quark densities are then found to satisfy the following equations.

$$\frac{du(x)}{d\log Q^2} = \alpha_\pi \int_x^1 \frac{dy}{y} [u(y)P_{uu}(z) + d(y)P_{ud}(z) + \pi^\circ(y)P_{u\pi^\circ}(z) + \pi^+(y)P_{u\pi^+}(z)] \quad (4.1)$$

$$\frac{d\bar{u}(x)}{d\log Q^2} = \alpha_\pi \int_x^1 \frac{dy}{y} [\bar{u}(y)P_{\bar{u}\bar{u}}(z) + \bar{d}(y)P_{\bar{u}\bar{d}}(z) + \pi^\circ(y)P_{\bar{u}\pi^\circ}(z) + \pi^-(y)P_{\bar{u}\pi^-}(z)] \quad (4.2)$$

and two similar equations for $d(x, Q^2)$ and $\bar{d}(x, Q^2)$. P_{ab} stands for the splitting function for $b \rightarrow a$ and $z = x/y$. The pion densities are found to satisfy

$$\frac{d\pi^+(x)}{d\log Q^2} = \alpha_\pi \int_\circ^1 \frac{dy}{y} [u(y)P_{\pi^+u}(z) + \bar{d}(y)P_{\pi^+\bar{d}}(z)] \quad (4.3)$$

$$\frac{d\pi^-(x)}{d\log Q^2} = \alpha_\pi \int_\circ^1 \frac{dy}{y} [d(y)P_{\pi^-d}(z) + \bar{u}(y)P_{\pi^-u}(z)] \quad (4.4)$$

$$\frac{d\pi^\circ(x)}{d\log Q^2} = \alpha_\pi \int_\circ^1 \frac{dy}{y} [u(y)P_{\pi^\circ u}(z) + d(y)P_{\pi^\circ d}(z) + \bar{u}(y)P_{\pi^\circ \bar{u}}(z) + \bar{d}(y)P_{\pi^\circ \bar{d}}(z)] \quad (4.5)$$

The splitting functions are given by

$$P_{q'q}(z) = \frac{1}{2} \frac{1+z^2}{1-z}, \quad P_{\pi q}(z) = 2z$$

$$P_{q'\pi} = \frac{1}{2} \left[(z^2 + (1-z)^2) + \left(\frac{e_\pi}{e'_q} \right)^2 z(1-z) \right] \quad (4.6)$$

The splitting functions obviously satisfy the requirement of charge conjugation invariance : $P_{qq} = P_{\bar{q}\bar{q}}$, $P_{q\pi^+} = P_{\bar{q}\pi^-}$, $P_{q\pi^\circ} = P_{\bar{q}\pi^\circ}$ etc. As given in (4.6), they do not satisfy the fermion number conservation, but this could be ensured by the usual regularization procedure.

5. Evolution of the nonsinglet distribution.

The first thing one notes about the results given above is that even in the lowest order the evolution of individual quark densities is flavor-dependent. However,

this alone can not explain the observed evolution of the Gottfried sum. Firstly, the first order calculation of the cross sections will always lead to $P_{\bar{q}q} = P_{q\bar{q}} = 0$. The contribution of $P_{q'\pi}$ drops out from the evolution of the nonsinglet distribution, and hence the dependence of $P_{q\pi}$ does not matter at all in this context.

Secondly, the drastic large Q^2 approximation used to obtain the results of the previous section will not lead to any evolution of the anomalous dimensions obtained by taking moments of the splitting functions. Hence, the Gottfried sum remains unaffected as it is controlled by the anomalous dimension for the first moment.

Therefore, in this model it is essential to consider both higher order perturbation terms and low Q^2 corrections. The splitting function $P_{\bar{q}q}$ is generated by the second order process and is seen to be of the form

$$P_{\bar{q}q}(z) \simeq \alpha_\pi \left[\frac{1}{2}(1 - z^2) + \left(\left(\frac{e_\pi}{e_{\bar{q}}} \right)^2 - 2 \right) (z(1 - z) + z^2 \log z) \right] \quad (5.1)$$

and gives a contribution to the anomalous dimension of the first moment

$$\gamma_1^{\bar{q}q} \simeq \frac{2\alpha_\pi}{9} \left[1 + \frac{1}{4} \left(\frac{e_\pi}{e_{\bar{q}}} \right)^2 \right] \quad (5.2)$$

This then points to a detectable evolution of the nonsinglet first moment and, hence, of the Gottfried sum. The anomalous dimension (5.2), obtained by retaining the large Q^2 approximation, is not dependent on Q^2 and, therefore, cannot conform to the observed departure from the Gottfried sum rule. The Q^2 dependence appears when one considers the low Q^2 corrections.

6. Conclusion.

The simple chiral potential model considered here leads to a nontrivial evolution of the nonsinglet moments even in the large Q^2 approximation. This is a nonperturbative phenomenon in QCD. The phenomenological model arrives at it by allowing for emission of an isovector meson by a quark. The detailed results for the evolution of the Gottfried sum with low Q^2 corrections will be reported elsewhere.

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